Eur. Phys. J. C 5, 357–361 (1998)
DOI 10.1007/s100529800853

THE EUROPEAN
PHYSICAL JOURNAL C

Comment on entropy production in nuclear collisions

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Received: 11 November 1997 / Published online: 10 March 1998

Abstract. A recent analysis of the data for particle production in central nucleus-nucleus (AA) collisions finds an "enhancement" in particle multiplicities or entropy at the highest energies near 200A GeV (as compared to those at lower energies). This is interpreted within a relativistic photon gas model as an increase in the number of degrees of freedom, and the formation of the quark-gluon plasma between AGS and SPS energies is hypothesized. We find that particle multiplicities in pA collisions also show an enhancement at large F, a nonlinear increase with the Fermi energy variable, F. This suggests the possibility that the enhancement seen in AA collisions is also due to such a non-linearity in F.

1 Introduction

Recently the analysis of NA35 and other particle production data from nucleon-nucleon, proton-nucleus and nucleus-nucleus collisions has raised a number of interesting questions [1–4]. In particular, Gaździcki has discussed the entropy produced in central nucleus-nucleus collisions [2]. He concludes that the enhancement of entropy (i.e., particle) production in central S+S collisions at 200A GeV (as compared to that near 12A GeV) may be interpreted as the manifestation of an increase, by about a factor of three, in the effective number of degrees of freedom in the early stages of the collision. The transition from hadronic to partonic degrees of freedom, and the formation of a quark-gluon plasma (QGP) at some energy between that of the AGS and the SPS is hypothesized.

It was Fermi [5] who first applied statistical methods to explain and predict particle (mainly pion) production in high energy nucleon-nucleon (NN) collisions. He pictured a collision as an event in which "the nucleons with their surrounding retinue of pions hit against each other so that all the portion of space occupied by the nucleons and by their surrounding pion field will be suddenly loaded with a great amount of energy". He argued that the strong interactions would rapidly distribute this energy among the various degrees of freedom, according to statistical laws.

Fermi also introduced the idea that, at extremely high energies, one could use thermodynamics and the idea of a relativistic photon gas where energy density, ε, is proportional to fourth power of temperature: $\varepsilon \sim T^4$, as in Stefan's law. At very high temperatures, in analogy to the photon case, multiple (massless) pion production was assumed with pion density proportional to T^3 or to $\varepsilon^{3/4}$.

It was Pomeranchuk [6] who argued that particles only really materialize, not at the early collision stage, but at the lower energy densities of later stages where one has freeze-out and "free separation" of particles. Landau [7] developed further these ideas, applying hydrodynamics of an ideal fluid to the high density, high temperature (expanding) system. He assumed a photon gas equation-of-state, $p = \varepsilon/3$ (p = pressure). This leads to $T\sigma = \varepsilon + p = 4\varepsilon/3$ and $\sigma \sim \varepsilon^{3/4}$ where σ is the entropy density. At an early stage the total entropy is $S_E = \sigma V$, where V is the relativistically contracted collision volume, $V \sim V_0/\sqrt{s_{NN}}$. The collision energy density available (for particle production) is taken to be [2]

$$\varepsilon = (\sqrt{s_{NN}} - 2m_N)/V \sim (\sqrt{s_{NN}} - 2m_N) \cdot \sqrt{s_{NN}} \ . \quad (1)$$

Thus the proportionality

$$S_E \sim (\sqrt{s_{NN}} - 2m_N)^{3/4}/(\sqrt{s_{NN}})^{1/4} \equiv F \ . \quad (2)$$

F is called the Fermi energy variable. Landau assumed adiabaticity (no shock waves during expansion and breakup, so entropy is virtually unchanged) and showed that ⟨particle production⟩ ∼ F even for a nonuniform system. It has been shown [2], except for low energies, that measured pion multiplicities, $\langle\pi\rangle$, in NN collisions are (as predicted by the Fermi/Landau model), proportional to F [2], up to $\sqrt{s_{NN}} \simeq 20$ GeV. At higher energies, the multiplicities increase more slowly with energy, approximately as $\sqrt{F}$ [8]. This breakdown of the Fermi model is probably due (largely) to the fact that there is not total stopping, but that appreciable energy goes into leading particles.

Landau extended the analysis to AA collisions with the result that S ∼ ⟨hadron multiplicty⟩ ∼ $A \cdot F$ for head-on collisions, in agreement with experimental data [1].

It is usually assumed that in the expansion following the early stage the (total) entropy, S, does not change much from its early stage value, S_E. At SPS and lower energies approximately 90% of the particles produced are pions and so S is approximately proportional to $\langle\pi\rangle$ or to the equivalent pion multiplicity [2].

The analysis of entropy production in central nucleus-nucleus collisions has been carried out in [2–4] using the

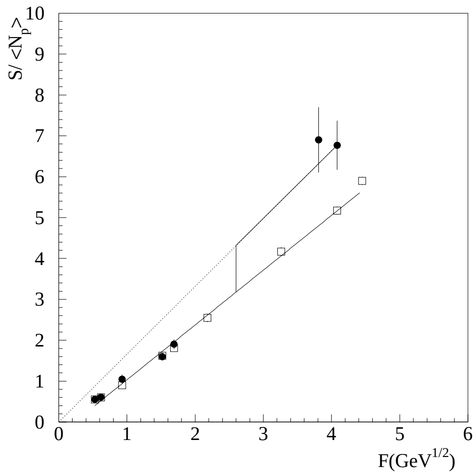

Fig. 1. $S/<N_p>$ is plotted vs. F for NN (*open squares*) and central AA (*closed circles*) collisions, including the NA49 preliminary point at F=3.81

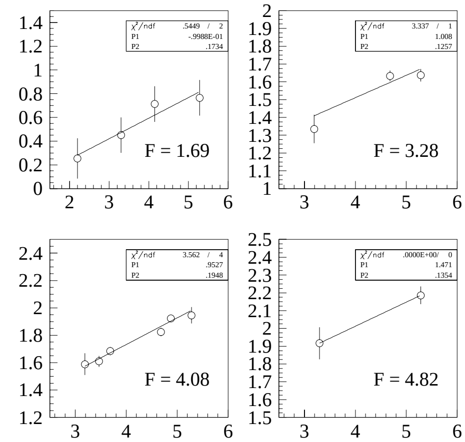

Fig. 2. ln $< h^- >_{pA}$ is plotted vs ln A. $\alpha(F)$ is determined for F=1.69-4.82 assuming $< h^- >_{pA} = I_0 A^\alpha$

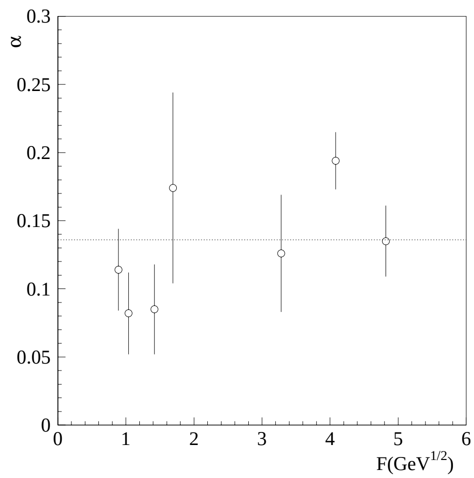

Fig. 3. α as determined at various F values. The *dashed line* is the average value of α

Fermi-Landau approach outlined above for the NN case. The data used are compiled in [1]. The lowest order approximation used [2] is $S_E \sim S \sim \langle\pi\rangle \sim \langle N_p\rangle \cdot F$, where $\langle N_p\rangle$ is the average number of participating nucleons. $\langle N_p\rangle$ depends on the centrality of the collision, i.e., what fraction of the total reaction cross-section (σ_R) is used in triggering the events used in the analysis. For central collisions, typical values used are 2-5% of σ_R. Experimentally, the $\langle N_p\rangle$ values used correspond approximately to the number of nucleons outside of the Fermi momentum spheres ($p \lesssim 300$MeV/c) of the projectile and target nuclei [1], as determined for projectile nuclei via measurements in a "veto" or "zero-degree" calorimeter.

Besides that in the pions, entropy is also contained in the other particles produced (mainly kaons) and is also transferred to the participant nucleons in the form of increased energy. Thus in [2] the total entropy, in units of entropy per pion, is estimated using

$$S = 3 \cdot (\langle h^-\rangle - \langle K^-\rangle) + k \cdot \langle K/\bar{K}\rangle + \delta \cdot \langle N_p\rangle \ . \qquad (3)$$

The first term estimates the entropy due to the pions; the second that carried by the kaons ($K^\pm, K^0, \bar{K}^0$), and the third that by the nucleons. From the difference between $\langle \pi/N_p\rangle_{AA}$ and $\langle \pi/(N_p = 2)\rangle_{NN}$ data at lower energies (AGS and below), δ is determined [2] to be 0.35.

In Fig. 1, $S/\langle N_p\rangle$ is plotted vs. F for NN (N_p=2) and central AA collisions. This plot is essentially Fig. 5 of [2] with F instead of F_{NN} [$F_{NN}(m_\pi = 0) = F$] and with the addition of preliminary SPS data from [9–10] for 158A GeV Pb+Pb collisions ($F = 3.81$). Relative to the (nearly linear) trend of the NN and the lower-energy AA data, the SPS 158A GeV Pb+Pb and 200A GeV S+S data show an enhancement of entropy production, and lie on a line of steeper slope. In the spirit of the Fermi/Landau approach used in [2] it is natural to interpret this as an "unusual increase of the entropy density at the early stage" of the collision, and further, to assume that this is due to an increase in the effective number of degrees of freedom (g) as is expected in the creation of deconfined (partonic) matter. Assuming $S \sim g^{1/4} \cdot \langle N_p\rangle \cdot F$, as in [2], the factor of 1.33 increase in slope, as given by the straight lines in Fig. 1, implies an increase of about a factor of 3 in g, the effective number of degrees of freedom.

2 Proton-nucleus data

While the above interpretation as given in [2] and [4] is certainly plausible, it hinges upon several assumptions: e.g., that the dependence of $\langle h^-\rangle_{AA}$ or $\langle\pi\rangle$ on F is linear, and that $\langle N_p\rangle$ is known at each F. Here, we focus on the multiplicity dependence and present evidence that in pA collisions the F dependence of particle production multiplicities, $\langle h^-\rangle_{pA}$, appears to be nonlinear.

The pA multiplicity data are taken from the compilation in [1]. In order to combine data for different A, we assume, at each F, that $\langle h^-\rangle_{pA} = I_0 A^\alpha$. A value of α is determined from a linear least squares fit of ln$\langle h^-\rangle_{pA}$ vs. ln A. Figure 2 shows fits for 14.6, 100, 200 and 360 GeV.

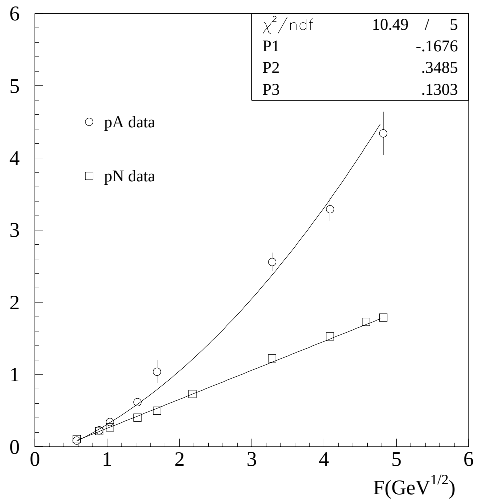

Fig. 4. Shown is $< h^- >_{pA}/A^{<\alpha>}$ vs F for pA data and $< h^- >_{pN}$ /2 vs F for pN data. The pN data is fit well with a straight line but the pA data is fit much better with a 2nd order polynomial with some F^2 dependence

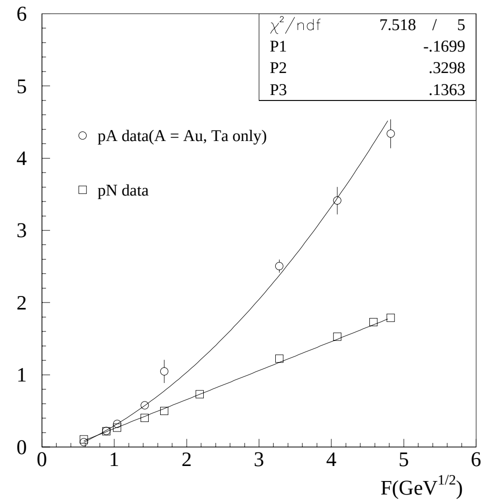

Fig. 5. Shown is $< h^- >_{pA}/A^{<\alpha>}$ vs F for pA data(A = Au, Ta only) and $< h^- >_{pN}$ /2 vs F for pN data. The pN data is fit well with a straight line but the pA data is fit much better with a 2nd order polynomial with some F^2 dependence

The values of I_0(F) given by the intercepts are found to be approximately equal to (to within 14%) the $\langle h^-\rangle_{pN}$ values which are determined for each F as the average of $\langle h^-\rangle_{pp}$ and $\langle h^-\rangle_{pn}$ data. Then (see later) A^α can be interpreted as the average number of collisions. Figure 3 shows the values of α found from the fits. A weighted average yields $\langle\alpha\rangle = 0.136$. For $F > 1.5$, $\langle\alpha\rangle \equiv \langle\alpha_h\rangle = 0.16 \pm .02$.

The pA multiplicity data at each energy (each F) are normalized via $R_{pA} = \langle h^-\rangle_{pA}/A^{\langle\alpha\rangle}$, and then averaged and plotted vs. F in Fig. 4. Also shown are the $\langle h^-\rangle_{pN}/2$ multiplicities, where the factor 1/2 is used to separate the data loci. A fit to the pA data using the form $a+bF+cF^2$ yields a much better χ^2/ndf than does one with $c = 0$.

The pN data are well fit by a straight line, while the pA multiplicities appear to increase with F in a nonlinear manner. This could be due to the possibility that at higher incident energies both the incident and the struck target nucleon in successive "thermalizing" collisions in the target are more effective in producing particles than at lower incident energies where successive collisions are closer to or below threshold. The possibility that at higher energies more target nucleons are struck seems unlikely since the value of α does not, within uncertainties, increase with F beyond $F = 1.69$ (14.6 GeV/c). The "suppression" of particle production at $F < 2$ in AA relative to NN collisions has been discussed in [1] and [16], and here we see evidence for this in pA collisions. See Figs. 4 and 5 where $\langle h^-\rangle_{pA}/2$ and $\langle h^-\rangle_{NN}$ are compared.

To minimize the uncertainties due to those in α we also use just the $\langle h^-\rangle_{pA}$ values for Ta and Au, normalized by the factor $A^{0.134}$, and plot these vs. F in Fig. 5. The quadratic fit is quite good with $\chi^2/ndf = 7.52/5 = 1.5$.

3 Discussion

The proportionality between S and F was derived in the Fermi/Landau approach(relativistic gas and hydrodynamical expansion) for symmetric systems (originally for NN, but extended to AA by Landau[7]). The energy density (1) is calculated assuming total stopping of the participants in the NN or AA cm system. It is now well known that in both systems, as well as in pA, stopping is incomplete – the nucleons retain some fraction of their initial cm energy. For example, near 200 A·GeV(lab) the mean rapidity shift in NN is $\Delta y \simeq 1.2$ units [11, 12], and in AA (^{32}S and ^{208}Pb) Δy appears to be $\simeq 1.6$ units [13, 14]. So, in the NN case the incoming nucleons in the cm system at y = ± 3 each lose on average $\simeq$ 5.0 GeV or $\simeq$ 51% of their energy. (We use $E = m_T \cosh y$ with $p_T \simeq 0.45$ GeV/c for NN, and do a rough integration over y.) For central AA(^{208}Pb) collisions at 158 A·GeV the mean loss is 5.5 GeV per nucleon or $\simeq$ 64%. (Here, we sum $m_T(y)\cosh y$ over y; $\langle p_T\rangle \simeq 0.85$ GeV/c.) The pA minimum bias values for pAg and pAu are near $\Delta y \simeq 2.0$ and $\langle E_{loss}\rangle$ = 7.4 GeV or $\simeq$ 75% of the incoming proton's cm energy. (These values have rather large uncertainties ($\simeq$ 10%).) At the AGS the 11.6A GeV Au+Au data indicate $\langle E_{loss}\rangle$ = 41%, while for NN it is 36% of E_{inc} in the cm system. The point is that relative to the total stopping assumed in the Fermi/Landau model, the experimental values indicate incomplete stopping which is energy dependent and system dependent. The fractional energy loss in AA collisions appears to increase with energy (AGS to SPS) and could explain the increased (AA) hadronic particle production measured [1].

The fact that the pA rapidity shift is larger than that apparent in AA is surprising. Compared to pA, there are (in central AA collisions) more "projectile" nucleons which have to traverse the thicker central region of the "target" nucleus. Possibly the AA rapidity shifts, which are difficult to determine from the relatively flat dN/dy distributions, are larger than they appear? Or the smaller AA stopping may be due to the fact that trailing nucleons encounter “wounded” as well as “fresh” oncoming nucleons, and collisions with the former will be at reduced cm energies, and may, as well, be less effective in stopping.

The above discussion makes it clear that despite the uncertainties the energy loss (or energy deposited) by the incident baryons is qualitatively similar, but quantitatively not the same, for NN, pA and AA. In the extension from NN to AA Landau [7] argued that the distance between nucleons in nuclei is essentially equal to the range of interaction. So, in the NN cm frame, the mass and energy density of colliding (and completely stopped) nuclei will be roughly the same as in the case of colliding (and stopped) nucleons at the same incoming velocities(same energy per nucleon). Of course, in this model the total AA energy and volume are A times larger.

The case of pA is different in that it lacks symmetry, so one needs a microscopic approach. In the NN cm frame the proton and nucleus (bag of nucleons) approach each other with equal and opposite velocities. Depending on impact parameter the proton makes, on average, one, two or three, etc, pN collisions, thus producing one or several regions of high energy density and particle production. Ignoring the effect of Fermi motion, the first collision takes place at the NN(pN) cm energy $E_{cm} = \sqrt{s_{NN}} \simeq 19.3$ GeV for 200 GeV lab. The corresponding F is 4.08 $\sqrt{GeV}$. Each nucleon loses on average $\simeq 51\%$ of its energy. The next collision of the “wounded” nucleon is head-on with one of the A nucleons having full energy in the original NN cm frame. The cm energy of this second collision is $E'_{cm} \simeq 13.8$ GeV and $F' = 3.3 = 0.81$ F, and so on. (Klar and Hüfner[11] have carried out a similar analysis.)

To illustrate the effect of the reduced values of F we assume for simplicity three successive pN collisions. Then F' of the second collision is a good estimate of $\langle F \rangle$ for the three. E.g., at 200 GeV (above) $\langle F \rangle = F' = 0.81\mathrm{F} = 3.3$. In Fig. 6 we plot $\langle h^- \rangle_{pA}/A^\alpha$ vs this reduced $\langle F \rangle = F'$. Also plotted is $\langle h^- \rangle_{pN}$ vs F (not $\langle h^- \rangle_{pN}/2$ as in Figs. 4 and 5). ($\langle F \rangle =$ F for NN collisions.) One sees that the $\langle h^- \rangle_{pA}$ nonlinearity remains. Relative to $\langle h^- \rangle_{pN}$ it appears that the nonlinearity is due more to an enhancement in $\langle h^- \rangle_{pA}$ at high energies ($\geq$ 15 GeV/c) than to a suppression at low energies.

There is a process that appears to contribute substantially more particle production to $\langle h^- \rangle_{pA}$ at higher energies (F values) than at low. In pA, as viewed from the pN(NN) cm system, the struck target nucleon (as well as the incident p) is slowed in the collision. Then target nucleons coming from behind with full cm energy can collide with the (slower) wounded target nucleon. Of course, these “secondary” collisions are at lower cm energies than those where the incident wounded nucleon hits an oncoming target nucleon. For incident energies below 20 GeV/c (F $\leq$ 2) pion production from these secondary collisions is on average negligible. However, at higher incident energies these secondary collisions between wounded and “fresh” target nucleons take place at cm energies (and F values) which produce appreciable $\langle \pi \rangle$. At 200 GeV/c incident, this production is estimated to be $\simeq 0.95 \pm 0.45\ \pi^-$ per pA collision, or about half a unit of $\langle h^- \rangle_{pA}/A^\alpha$ in Fig. 5 at F = 4.08, and in Fig. 6 at $\langle F \rangle = 3.3$. (We assume the linear relation between $\langle h^- \rangle_{NN}$ and F given in [1] and shown in Fig. 1.) Thus it seems likely that these secondary collisions can, at higher incident energies, contribute to both $\langle h^- \rangle_{pA}$ and $\langle h^- \rangle_{AA}$ and produce some nonlinearity in F.

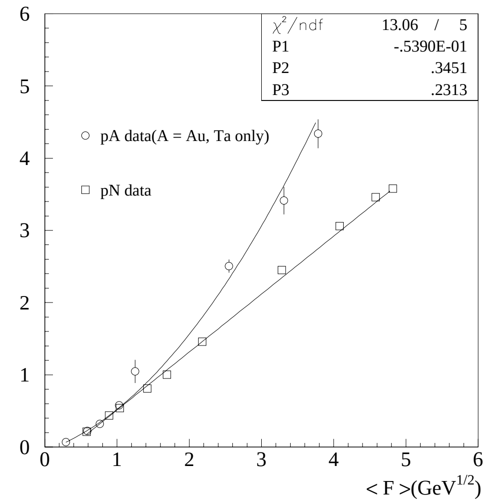

Fig. 6. Shown is $< h^- >_{pA}/A^{<\alpha>}$ vs $\langle F \rangle$ for pA data(A = Au, Ta only) and $< h^- >_{pN}$ vs F ($\langle F \rangle =$ F for pN collisions) for pN data. The non-linearity of the pA data is hardly changed by using $\langle F \rangle$, and the pA and pN data agree well at low F

The suppression of pion production in nuclear (AA) collisions at lower energies (2–15 A· GeV/c), which gives rise to the δ in (3), has been discussed in [16]. The authors suggest that the suppression is due mainly to $\Delta + N \to NN$ in the equilibration process. This reaction does not appear to have an energy dependence that would explain the relative enhancement of $\langle h^- \rangle_{AA}$ at higher energies (Fig. 1). Charge exchange reactions, such as $\pi^- p \to n\pi^0$, have quite a strong energy dependence which would tend to suppress charged pion production more at low than at high incident energy. Our pA analysis indicates that the “suppression” of pion production at lower F values could be due to the reduction of the effective (collision) F value in successive collisions. Thus in Fig. 6, where we use a mean F, $\langle F \rangle$, $\langle h^- \rangle$ from pN and pA roughly agree at low $\langle F \rangle$.

The notion that pA(as well as AA) collisions can be modelled as successive NN collisions is arguable. As noted earlier, this notion is supported by the fact that pA and pN multiplicity data are rather well-described by $\langle h^- \rangle_{pA} = I_0 A^\alpha$, with $I_0 \simeq \langle h^- \rangle_{pN}$. Naively, one expects $\langle h^- \rangle_{pA} = \langle h^- \rangle_{pN}\ \overline{\nu}(pA)$, where $\overline{\nu}$ is the average number of target nucleons hit by the incident proton. Then $\overline{\nu}(pA) = A^\alpha$ in $I_0 A^\alpha$. One estimate is $\overline{\nu} = A\ \sigma_{in}(pN)/\sigma_{in}(pA)$, where σ_{in} is the inelastic cross section. Data in this energy range give $\sigma_{in}(pA) \simeq \sigma_0 A^{\alpha_o}$ with $\alpha_o = 0.72 \pm 0.05$[15]. For $\alpha_o = 0.72$ one calculates $\overline{\nu}(pA) \sim A^{\alpha'}$ with $\alpha' = 0.28 \pm 0.05$. High energy data yield $\overline{\nu} \sim A^{\alpha'}$ with $\alpha' = 0.21 \pm 0.05$ [15], which agrees with our extracted $\langle \alpha_h \rangle = 0.16 \pm .02$.

On the theoretical side, Landau [7] and Belen'kii and Milekhin [17] have considered pA collisions. They find that a good approximation(to $\leq 4\%$) is $S = S_0 A^{0.19}$ or $\langle h \rangle_{pA} = \langle h \rangle_{pN} A^{0.19}$ where S_0 and N_{NN} are, respectively, the entropy and number of particles formed in a NN collision. Again, this agrees, within uncertainties, with α' and $\langle \alpha_h \rangle$.

In Fig. 3 there appears to be a decrease of α as one goes to low energies. This is expected in pA since in the limit of very low energies only the first collision of the proton with a "target" nucleon in A will be effective in producing particles; for $\alpha \to 0$, $A^{\alpha} \to 1$.

4 Summary

The use of F to characterize particle production multiplicities, e.g., via $\langle \pi \rangle \sim F$ in NN and AA collisions, assumes that there is complete stopping or that the stopping (energy loss) is approximately the same fraction at all energies and for all systems. However, the available data on stopping indicate that the fractional stopping or energy loss per nucleon is energy and system dependent; i.e. it is not the same for NN, pA and AA systems. In a macroscopic Fermi/Landau-type model the increase in the fractional energy loss from AGS to SPS energies would contribute to the nonlinear increase in $S/\langle N_p \rangle$ (Fig. 1).

The data on pA multiplicities, $\langle h^- \rangle_{pA}$, appear to show a nonlinear increase with the Fermi energy variable, F. A simple NN collision model suggests that at higher (SPS) energies secondary collisions between the struck ("wounded") and "fresh" target nucleons contribute to hadron production and to the nonlinearity. This should also apply to AA collisions.

Clearly it is possible to fit the AA data in Fig. 1 with a quadratic F dependence. We have no proof that this is the correct behavior; however, this possibility is suggested by the F dependence of the pA multiplicities. Experimental data at one or more intermediate energies (between AGS and SPS values) could determine whether the increase in $S/\langle N_p \rangle$ is continuous or takes a jump.

Acknowledgements. We acknowledge several interesting and helpful discussions with M. Gaździcki and the support of the NA49 Collaboration. This work is supported by the U.S. National Science Foundation, Grant #PHY-96-01271, and the University of California.

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